Asymmetric Impulsive Loading of Shallow Spherical Shells

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Theme

THE main purpose of the paper is to study the dynamic deformation characteristics of shallow spherical shells when subjected to asymmetric loadings. To study these characteristics, explicit solutions and numerical results are presented for the specific cases of impulsive concentrated moment and tangential force applied at the apex. On the basis of these numerical results, the influence of various physical and loading conditions on the transient responses are illustrated.

Contents

The forced response studies on shallow spherical shells have been carried out earlier by Naghdi and Orthowein,[1] Tsai,[2] and Reismann and Culkowski.[3,4] Their analytical and numerical results were restricted to distributed axisymmetric loadings. Recently, the present authors[5,6] have discussed axisymmetric response of shallow spherical shells with different types of normal loads, pulse shapes, and edge conditions. In the present paper, the previous work has been extended to include the asymmetric loading of shallow spherical shells.

The governing equations of shallow spherical shells using improved thin-shell theory may be written in the operator form

$$\begin{aligned} L_1(u,v,w) &= K_1 u_{,tt} - p_r^* \\ L_2(u,v,w) &= K_1 v_{,tt} - p_\theta^* \\ L_3(u,v,w,\beta_r,\beta_\theta) &= K_2 w_{,tt} - p_z^* \\ L_4(w,\beta_r,\beta_\theta) &= \frac{h^2}{12} K_r \beta_{r,tt} - m_r^* \\ L_5(w,\beta_r,\beta_\theta) &= \frac{h^2}{12} K_r \beta_{\theta,tt} - m_\theta^* \end{aligned} \tag{1}$$

where the operators L_1–L_5 are given in the full paper. The applied distributed forces are p_r^*, p_θ^*, and p_z^* and the applied distributed moments are m_r^*, m_θ^*. All quantities are in nondimensional form and the scheme of nondimensionalization is given in the full paper.

Using spectral representation, the solution to the asymmetric response is expressed as

$$\begin{Bmatrix} u(r,\theta,t) \\ v(r,\theta,t) \\ w(r,\theta,t) \\ \beta_r(r,\theta,t) \\ \beta_\theta(r,\theta,t) \end{Bmatrix} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \begin{Bmatrix} U_m(r)\cos n\theta \\ V_m(r)\sin n\theta \\ W_m(r)\cos n\theta \\ \beta_{1m}(r)\cos n\theta \\ \beta_{2m}(r)\sin n\theta \end{Bmatrix} q_{mn}(t) \tag{2}$$

where $q_{mn}(t)$ are the generalized coordinates and $U_m, \ldots, \beta_{2m}$ are shape functions of the normal modes which are again given in the full paper. The orthogonality conditions for the present problem are shown to be

$$\int_0^{2\pi}\int_0^1 \left\{\left[K_1 U_m U_i + K_2 W_m W_i + \frac{h^2}{12} K_r \beta_{1m}\beta_{1i}\right]\cos n\theta \cos j\theta + \left[K_1 V_m V_i + \frac{h^2}{12} K_r \beta_{2m}\beta_{2i}\right]\sin n\theta \sin j\theta\right\} r\,dr\,d\theta$$
$$\begin{aligned} &= 0 \quad \text{for} \quad m \neq i, n \neq j \\ &= M_{mn} \quad \text{for} \quad m = i, n = j \end{aligned} \tag{3}$$

The equations for the determination of the generalized coordinates are shown to take the following form

$$q_{mn,tt} + \omega_{mn}^2 q_{mn} = G_{mn}(r)F(t) \tag{4}$$

where, for the case of generally distributed loads

$$G_{mn}(r) = \frac{1}{a^2 M_{mn}}\int_0^{2\pi}\int_0^a [(p_r U_m + p_z W_m + m_r \beta_{1m})\cos n\theta + (p_\theta V_m + m_\theta \beta_{2m})\sin n\theta)]\bar{r}\,d\bar{r}\,d\theta \tag{5}$$

The solution to Eq. (4) is written as

$$q_{mn}(t) = G_{mn}(r)\int_0^t F(t')\sin\omega_{mn}(t-t')\,dt' \tag{6}$$

Values of the function $G_{mn}(r)$ for the various cases considered in the study are summarized as: 1) concentrated tangential force at the apex

$$G_{m1} = [P_r U_{m1}(0)/M_{m1}\omega_{m1}] \tag{7}$$

and 2) concentrated moment at the apex

$$G_{m1} = [M_r \beta_{rm1}(0)/M_{m1}\omega_{m1}] \tag{8}$$

where

$$P_r = \frac{\bar{P}_r(1-\nu^2)}{E\bar{h}a}, \qquad M_r = \frac{\bar{M}_r(1-\nu^2)}{E\bar{h}a^2}$$

Table 1 Contributions from individual modes to total normal displacement[a] at a point close to the apex [on $(\bar{r}/a) = 0.1$]

Sl. no.	Contributions to normal displacement: Due to normal load	Due to moment	Due to tangential load
1	−0.2454 E 02	−0.1803 E 02	0.5225 E 00
2	−0.2524 E 01	−0.5015 E 02	0.9233 E-02
3	−0.1835 E 02	−0.7730 E 02	0.4304 E 00
4	−0.9705 E 01	−0.4226 E 02	0.1584 E-02
5	−0.3200 E 01	−0.2734 E 02	0.1216 E 00
6	−0.2649 E 01	−0.5406 E 00[b]	−0.1148 E 00[b]
7	−0.1334 E 01	−0.2075 E 02	0.5855 E-01
8	−0.1021 E 00	−0.1531 E 02	−0.3930 E-02
9	−0.9621 E 02[b]	−0.1069 E-01[b]	0.7144 E-02[b]
10	−0.7656 E-01	−0.8910 E 01	0.7613 E-02
11	−0.5404 E 00	−0.5664 E 01	−0.2308 E-02
12	−0.3991 E-02	−0.2525 E-02[b]	0.2281 E-02[b]
13	−0.1559 E-02[b]	−0.4718 E-02[b]	0.6750 E-03[b]
14	−0.3210 E-02	−0.3163 E 01	−0.9379 E-05
15	−0.2530 E 00	−0.1124 E 01	0.5120 E-03
16	−0.2089 E-02	−0.1068 E-04[b]	−0.1007 E-03[b]
17	−0.7139 E-03[b]	0.3359 E-01	−0.1483 E-05
18	−0.1775 E-02	0.4570 E-02[b]	−0.1013 E-02[b]
19	−0.1533 E-02	0.1644 E-03[b]	−0.4359 E-03[b]

[a] Due to different types of load at the peak of a half sine pulse, with $T = 10$ ($h = 0.02$, $S = 0.144$, clamped edge).
[b] Represents contributions from longitudinal modes.

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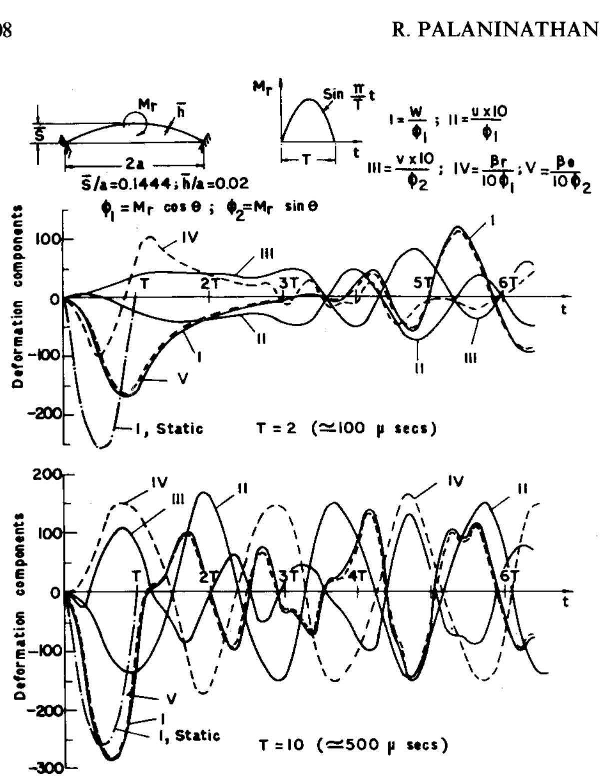

Fig. 1 Deformation components at radius $a/10$ for clamped edge under concentrated moment.

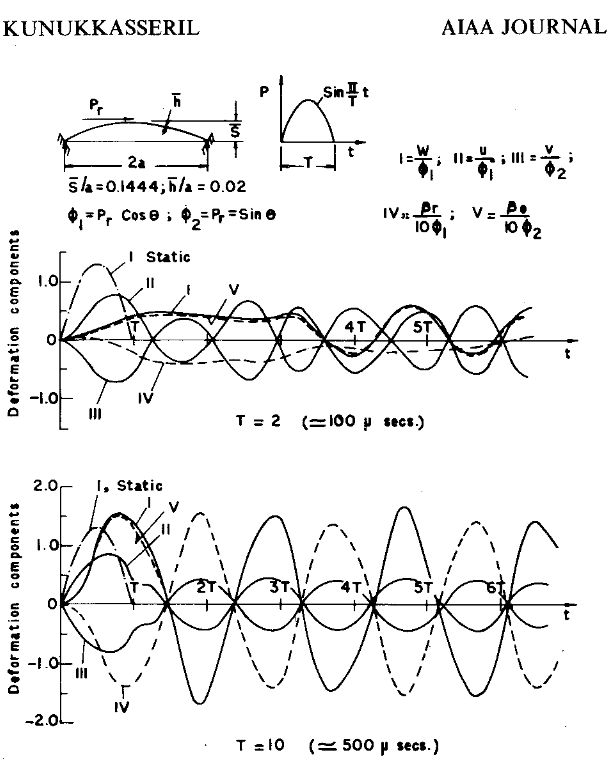

Fig. 2 Deformation components at radius $a/10$ for clamped edge under concentrated tangential force.

The pulse shape used in the analysis is that of a half sine pulse, and the explicit expressions for the generalized coordinates for this case are given in Ref. 6.

Extensive numerical results are presented for the two previous loading conditions. First using 20 modes with one nodal diameter, the transient responses are calculated for the clamped and simply supported edge conditions. These 20 modes are tabulated in the full paper for four shell models and the two edge conditions.

One of the interesting aspects which is of importance is to know the relative contributions from individual modes to the total normal displacement component due to the three types of load: 1) concentrated normal, 2) tangential, and 3) moment loads applied at the apex. From Table 1, it may be seen that in the determination of response due to normal load and moment, the longitudinal modes could be omitted, since the contribution from them is relatively smaller than that from the adjacent transverse modes. These contributions are about $\frac{1}{10}$ and $\frac{1}{100}$ of that of the adjacent transverse modes, respectively, for normal load and moment. Further, in the determination of response due to tangential load, the individual contributions from the two types of modes are of equal magnitudes, and hence both must be considered in the analysis.

Eight figures are presented in the full paper to illustrate the various dynamic features and the effects of physical conditions. The first six of these figures illustrate the variation of deformation components with respect to time. The last two figures show the displacement and strain across a meridional section at various instants. Two typical figures from these are presented here as Figs. 1 and 2. From these figures it is quite clear that for the shorter pulse duration ($T = 2$), the displacement components are lower than the static values. From Fig. 2 it may also be noted that for tangential force, the tangential displacement components are larger than the normal component w for $T = 2$. In general, for pulse durations of the order of natural periods of the first few modes, the magnitude of the displacement components is larger than the static values.

Some of the relevant conclusions that may be derived from the present study may be listed briefly. 1) There is no appreciable influence of the edge conditions on the impulsive responses considered here. 2) Contributions from the longitudinal modes can be omitted when normal load and moment are applied; however, these modes should also be included when tangential loads are applied. 3) Pulse durations of the order of half the periods of first few normal modes should be of concern to the structural designer.

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